COMPUTATIONAL FINANCE

Lecture 0: Review of Net Present Value
Simple Java NPV Applet

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Some *Deep Theoretical Insights* in Finance

1. More is better (dominance).
2. Something for nothing (an arb) is a good thing.
3. Mispricing represents a profit opportunity:
   (a) Buy it if it's too cheap.
   (b) Sell it if it's too expensive.
4. Plan your trades to convert a profit opportunity into an arb.
5. Something for nothing is not easy to find.

Net Present Value (NPV) Formula

Consider net cash flows C_0, C_1, ... , C_T, where C_0 is the cash flow now, C_1 is the cash flow to be received one year from now, C_2 is the cash flow to be received two years from now, etc. Given access to borrowing and lending at the riskless interest rate r, receiving the cash flows (C_0, C_1, ... , C_T) is worth the same as receiving at time 0 the NPV defined by

$$NPV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + ... + \frac{C_T}{(1+r)^T}.$$

What does this result have to do with our *Deep Theoretical Insights*?

NPV: the Arb in One Period

Suppose the cash flow (C_0, C_1) is selling in the market for P, where $P < NPV = C_0 + C_1/(1+r)$. Then we can construct an arb as follows. First, we know we should buy more because it is too cheap, so we will buy one unit in the market. (Of course, this can be scaled up until we start affecting the price, but all we need to do is to show that this is profitable at some scale.) Second, we need to plan our trades to convert a profit opportunity into an arb. We do this by borrowing enough (lending if C_1 is negative) now for repayment at time 1 to move all the cash flows to time 0.

Time 0	Time 1	
$-P + C_0$	C_1	purchase of the cash flow
$C_1/(1+r)$	$-C_1$	one-period riskless borrowing
$-P + C_0 + C_1/(1+r)$	0	total net cash flow: the arb

Questions: What is the correct NPV formula if we want the money out a year from now? What do we do if $P > NPV$?

Investment (Future Value)

Time	Wealth
0	W
1	$W(1+r)$
2	$W(1+r)^2$
3	$W(1+r)^3$
⋮	⋮
T	$W(1+r)^T$

Derivation: Investing from $t-1$ to t, we start with $W(1+r)^{t-1}$ and we get interest of $rW(1+r)^{t-1}$, for a total amount at t of $W(1+r)^{t-1}+rW(1+r)^{t-1} = (1+r)(W(1+r)^{t-1}) = W(1+r)^t$.

In-Class Exercise: NPV Arb

Suppose P is the market price of the cash flows C_0, C_1, ... , C_T, and that $P > NPV = C_0 + C_1/(1+r) + C_2/(1+r)^2 + ... + C_T/(1+r)^T$. Construct an arbitrage that takes advantage of this mispricing.

NPV Applet: NpvA.html

```
<HTML>
<HEAD>
<TITLE>A Simple NPV APPLET</TITLE>
</HEAD>
<BODY>
<APPLET CODE=NpvA.class WIDTH=400 HEIGHT=350>
</APPLET>
</BODY>
</HTML>
```

NPV Applet: NpvA.java

```
/* NPV Applet (a lite application to get started) */

import java.applet.*;
import java.awt.*;

public class NpvA extends Applet {
  TextField c0,c1,c2,c3,c4,c5,R;
  Label npvchar;
  public NpvA() {
    setLayout(new BorderLayout());
    add("North",new Label("Sample NPV Applet",Label.CENTER));
    Panel centr = new Panel();
      centr.setLayout(new FlowLayout());
      Panel leftcentr = new Panel();
        leftcentr.setLayout(new GridLayout(7,1));
        leftcentr.add(new Label("Cash Flows",Label.CENTER));
        Panel in0 = new Panel();
        in0.setLayout(new FlowLayout(FlowLayout.LEFT));
        in0.add(new Label("year 0"));
```

```
in0.add(c0 = new TextField("-100",12));
leftcentr.add(in0);
Panel in1 = new Panel();
in1.setLayout(new FlowLayout(FlowLayout.LEFT));
in1.add(new Label("year 1"));
in1.add(c1 = new TextField("5",12));
leftcentr.add(in1);
Panel in2 = new Panel();
in2.setLayout(new FlowLayout(FlowLayout.LEFT));
in2.add(new Label("year 2"));
in2.add(c2 = new TextField("5",12));
leftcentr.add(in2);
Panel in3 = new Panel();
in3.setLayout(new FlowLayout(FlowLayout.LEFT));
in3.add(new Label("year 3"));
in3.add(c3 = new TextField("5",12));
leftcentr.add(in3);
Panel in4 = new Panel();
in4.setLayout(new FlowLayout(FlowLayout.LEFT));
in4.add(new Label("year 4"));
in4.add(c4 = new TextField("5",12));
```

```
      leftcentr.add(in4);
      Panel in5 = new Panel();
      in5.setLayout(new FlowLayout(FlowLayout.LEFT));
      in5.add(new Label("year 5"));
      in5.add(c5 = new TextField("105",12));
      leftcentr.add(in5);
      centr.add(leftcentr);
  Panel rightcentr = new Panel();
    rightcentr.setLayout(new GridLayout(4,1));
    rightcentr.add(new Label("Interest Rate (%):"));
    rightcentr.add(R = new TextField("5",12));
    rightcentr.add(new Label("The NPV is:"));
    rightcentr.add(npvchar = new Label("",Label.LEFT));
    npvchar.resize(180,npvchar.size().height);
    centr.add(rightcentr);
  add("Center",centr);
  recalc();}
public boolean action(Event ev, Object arg) {
  if(ev.target instanceof TextField) {
    recalc();
    return true;}
```

```
  return false;}
double text2double(TextField tf) {
  return Double.valueOf(tf.getText()).doubleValue();}
public void recalc() {
  npvchar.setText(String.valueOf(npv(text2double(R)/100.0,
    text2double(c0),text2double(c1),text2double(c2),
    text2double(c3),text2double(c4),text2double(c5))));}
float npv(double r, double c0, double c1, double c2, double c3,
  double c4, double c5) {
  double disc = 1.0/(1.0+r);
  return((float) (c0 + disc*(c1 + disc*(c2 + disc*(c3
    + disc*(c4 + disc*c5))))));}}
```