FIN 524B Exam

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This is a closed-book examination. You may not use texts, notes, a crib sheet, calculator, cell phone, listening device, or any other electronics. Answer all questions as directed on the Blue Books provided. Be sure your answers are clearly marked. There are no trick questions on the exam, but you should read the questions carefully. Good luck!

0. PLEDGE

The work on this exam is my own alone, and I have conformed with the rules of the exam and the code of conduct of the Olin School.

Signed name ____

Printed name (write clearly) ____

1. **Short answer** Answer in no more than one sentence of ordinary length. (25 points)

A. What is the main assumption in option pricing models?

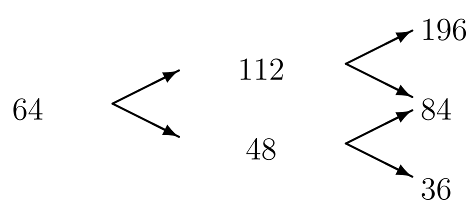
B. What are the two most important sources of randomness in traded equity put and call option prices?

C. When we price futures, why don't we discount the risk-neutral expected end-of-period payoff?

D. How can we interpret the term $SN(x_1)$ in the Black-Scholes call pricing formula?

E. Why might a company combine short-term borrowing with entering an interest rate swap?

2. **Binomial model: American and European puts** (30 points) Consider the binomial model with $u = 7/4$, $d = 3/4$, and $r = 5/4$. The stock price is initially \$64 and the stock is not expected to pay any dividends in the next few periods. The actual probabilities are 2/3 for the up state and 1/3 for the down state. Here is the stock price tree:



A. What are the risk-neutral probabilities?

B. Price a European put option on the stock with a strike price of \$61 and maturity two periods from now.

C. Price the corresponding American put option.

3. **Put-call Parity** (20 points) A stock with a price of \$40 has two listed options with strike equal to \$44 and a maturity a year from now, a put with a price of \$9 and a call with a price of \$10. The one-year risk-free rate is 10% simple interest. Assume interest rates will always be positive.

A. Show that put-call parity does not hold.

B. Suppose it is known that the stock will not pay any dividends during the year. Could the violation of put-call parity be due to the fact that these are American rather than European options? Explain briefly.

4. **Bond Option Pricing** (25 points) The risk-free inteerest rate starts at 50% and goes up or down by 25% each period. At all nodes, the actual probabilities are $\pi_u = 2/3$ and $\pi_d = 1/3$, and the risk-neutral probabilities at all nodes are $\pi_u^* = \pi_d^* = 1/2$.

A. Draw the interest rate tree.

B. Draw the price tree of a discount bond with face 105 and maturity two periods from now.

C. A European call option on the discount bond matures one period from now and has a strike price of 72. Draw the price tree for the option.

5. **Bonus problem** (20 bonus points) A stock has price \$70, the interest rate is 10% (simple interest), a call option maturing a year from now with strike price \$22 has price \$49, and a put option maturing a year from now with strike price \$33 has price \$2.20. Assume that the options are both American options, and that it is known that the stock will not pay any dividends this year. Show there is arbitrage by describing the strategy to follow.

Some Useful Formulas

Binomial model: if the stock has up factor u and down factor d, and one plus the riskfree rate is r, then the risk-neutral probabilities are:

$$\pi_u^* = \frac{r-d}{u-d}$$

$$\pi_d^* = \frac{u-r}{u-d}$$

and the one-period valuation is

$$\text{price} = \frac{1}{\text{r}}(\pi_\text{u}^* \text{V}_\text{u} + \pi_\text{d}^* \text{V}_\text{d}).$$

The Black-Scholes call price is

$$C(S,T) = SN(x_1) - BN(x_2),$$

where S is the stock price, $N(\cdot)$ is the cumulative normal distribution function, T is time-to-maturity, $B = Xe^{-rT}$ is the bond price, r is the continuously-compounded riskfree rate, σ is the standard deviation of stock returns (so σ^2 is the local variance),

$$x_1 = \frac{\log(S/B)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T},$$

and

$$x_2 = \frac{\log(S/B)}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T},$$

Note that $\log(\cdot)$ is the natural logarithm.

The Black-Scholes call price can be approximated by

$$\frac{S-B}{2} + 0.4\frac{S+B}{2}\sigma\sqrt{T}.$$

The put-call parity formula is

$$B + C = S + P.$$