FIN 524B Exam

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April 30, 2025

1. **Short answer** Answer in no more than one sentence of ordinary length. (25 points)

A. What is the main assumption in option pricing models?

absence of arbitrage

B. What are the two most important sources of randomness in traded equity put and call option prices?

stock price and vol

C. When we price futures, why don't we discount the risk-neutral expected end-of-period payoff?

Futures are not assets that transport value over time.

D. How can we interpret the term $SN(x_1)$ in the Black-Scholes call pricing formula?

The value of receiving the stock price when it is larger than the strike price. (Other answers are possible. For example, it is the value of the stock held in the replicating portfolio.)

E. Why might a company combine short-term borrowing with entering an interest rate swap?

to hedge interest rate risk but retain credit risk, which makes sense when the company's leaders think the market undervalues the company's credit

2. **Binomial model: American and European puts** (30 points) Consider the binomial model with $u = 7/4$, $d = 3/4$, and $r = 5/4$. The stock price is initially \$64 and the stock is not expected to pay any dividends in the next few periods. The actual probabilities are 2/3 for the up state and 1/3 for the down state. Here is the stock price tree:

$$\begin{tikzcd}[row sep=small] & & 196 \\ & 112 \arrow[ur] \arrow[dr] & \\ 64 \arrow[ur] \arrow[dr] & & 84 \\ & 48 \arrow[ur] \arrow[dr] & \\ & & 36 \end{tikzcd}$$

A. What are the risk-neutral probabilities?

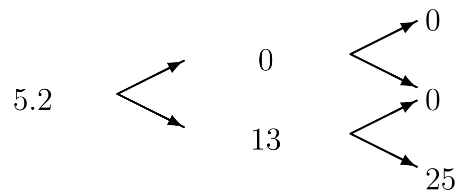
$\pi_u^* = \frac{r-d}{u-d} = \frac{5/4-3/4}{7/4-3/4} = \frac{1}{2}$
$\pi_d^* = 1 - \pi_u^* = \frac{1}{2}$

B. Price a European put option on the stock with a strike price of \$61 and maturity two periods from now.

$$\begin{tikzcd}[row sep=small] & & 0 \\ & 0 \arrow[ur] \arrow[dr] & \\ 4 \arrow[ur] \arrow[dr] & & 0 \\ & 10 \arrow[ur] \arrow[dr] & \\ & & 25 \end{tikzcd}$$

For example, $(1/r)(\pi_u^* V_u + \pi_d^* V_d) = (4/5)((1/2) \times 0 + (1/2) \times 25) = 10$

C. Price the corresponding American put option.



For example, $\max(61-48,(4/5)((1/2)\times 0+(1/2)\times 25) = max(13,10) = 13$
And, $max(61-64,(4/5)((1/2)\times 0+(1/2)\times 13) = 5.2$

3. **Put-call Parity** (20 points) A stock with a price of \$40 has two listed options with strike equal to \$44 and a maturity a year from now, a put with a price of \$9 and a call with a price of \$10. The one-year risk-free rate is 10% simple interest. Assume interest rates will always be positive.

A. Show that put-call parity does not hold.

Put-call parity says $S+P=B+C$. However, $S+P=40+9=49$ and $B+C=44/1.1+10=50$.

B. Suppose it is known that the stock will not pay any dividends during the year. Could the violation of put-call parity be due to the fact that these are American rather than European options? Explain briefly.

No. The American put can be worth more than the European, but the American call is worth the same as the European call (since early exercise is dominated if there are no dividends). This could explain $S+P>B+C$, but we have $S+P<B+C$.

4. **Bond Option Pricing** (25 points) The risk-free inteerest rate starts at 50% and goes up or down by 25% each period. At all nodes, the actual probabilities are $\pi_u = 2/3$ and $\pi_d = 1/3$, and the risk-neutral probabilities at all nodes are $\pi_u^* = \pi_d^* = 1/2$.

A. Draw the interest rate tree.

$$\begin{tikzcd}
 & & 100\% \\
 & 75\% \arrow[ur] \arrow[dr] & \\
50\% \arrow[ur] \arrow[dr] & & 50\% \\
 & 25\% \arrow[ur] \arrow[dr] & \\
 & & 0\%
\end{tikzcd}$$

B. Draw the price tree of a discount bond with face 105 and maturity two periods from now.

$$\begin{tikzcd}
 & & 105 \\
 & 60 \arrow[ur] \arrow[dr] & \\
48 \arrow[ur] \arrow[dr] & & 105 \\
 & 84 \arrow[ur] \arrow[dr] & \\
 & & 105
\end{tikzcd}$$

For example, $(1/r)((1/2) \times 105 + (1/2) \times 105) = (1/1.75)(1/2) \times 105 = (4/7)(1/2) \times 105 = 60$ And, $(1/1.5)((1/2) \times 60 + (1/2) \times 84) = (2/3) \times 72 = 48$.

C. A European call option on the discount bond matures one period from now and has a strike price of 72. Draw the price tree for the option.

$$\begin{tikzcd}
 & 0 \\
4 \arrow[ur] \arrow[dr] & \\
 & 12
\end{tikzcd}$$

$(1/1.5)((1/2) \times 0 + (1/2) \times 12) = (2/3) \times 6 = 4$