FIN 524B Exam answers

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1. **True-False** (25 points)

A. Absence of arbitrage is the main concept underlying the binomial option pricing model.

TRUE.

B. When volatility increases, the call price increases and the put price falls.

FALSE. The call price and the put price both increase.

C. When interest rates are positive, it is always dominated to exercise an American call option on a dividend-paying stock before maturity.

FALSE. You may want to exercise before the stock value is expected to be paid out in a dividend.

D. It is rare to pay off a mortgage when mortgage interest rates are higher than the existing mortgage's interest rate.

FALSE. There are many reasons for paying off a mortgage, for example selling the house to buy another, for which the interest rate is not the primary consideration.

E. A company borrowing money can borrow floating and use an interest rate swap to separate exposure to interest risk from exposure to the firm's credit risk.

TRUE. A fixed-for-floating swap hedges the interest rate risk without changing the credit exposure.

2. **Binomial model: futures option** (30 points) Consider the binomial model with $u = 2$, $d = 1/2$, and $r = 5/4$. The stock price is initially \$100 and the stock is not expected to pay any dividends in the next few periods. The actual probabilities are 2/3 for the up state and 1/3 for the down state.

A. What are the risk-neutral probabilities?

$\pi_u^* = \frac{r-d}{u-d} = \frac{5/4-1/2}{2-1/2} = \frac{3/4}{3/2} = \frac{1}{2}$
$\pi_d^* = 1 - \pi_u^* = \frac{1}{2}$

B. What is the futures price for a stock futures maturing two periods from now?

Underlying stock

$$
\begin{tikzcd}
 & & 400 \\
 & 200 \arrow[ur] \arrow[dr] & \\
100 \arrow[ur] \arrow[dr] & & 100 \\
 & 50 \arrow[ur] \arrow[dr] & \\
 & & 25
\end{tikzcd}
$$

Futures

$F = \pi_u^* F_u + \pi^* d F_d$

$$
\begin{tikzcd}
 & & 400 \\
 & 250 \arrow[ur] \arrow[dr] & \\
151.25 \arrow[ur] \arrow[dr] & & 100 \\
 & 62.5 \arrow[ur] \arrow[dr] & \\
 & & 25
\end{tikzcd}
$$

C. Price a European put option on the futures with a strike price of \$100 one period from now.

$\max(100 - 250, 0) = 0$, $\max(100 - 62.5, 0) = 37.5$

$$\begin{tikzcd}[row sep=small] & 0 \\ 15 \arrow[ur] \arrow[dr] & \\ & 37.5 \end{tikzcd}$$

3. **Put-call Parity** (20 points) A stock with a price of \$70 has two listed options with strike equal to \$77 and a maturity a year from now, a put with a price of \$12 and a call with a price of \$10. The one-year risk-free rate is 10% simple interest. Assume interest rates will always be positive.

A. Show that put-call parity does not hold.

Asking whether $S + P = B + C$

$S + P = 70 + 12 = 82$, but $B + C = 70 + 10$

B. Suppose it is known that the stock will not pay any dividends during the year. Could the violation of put-call parity be due to the fact that these are American rather than European options? Explain briefly.

YES. Given positive interest rates and no dividends, the American call is worth the same as a European call, but the American put can be worth more than the European put. The American put is worth 12; maybe the European put is worth 10 so put-call parity holds for European options, as it should.

4. **Approximate Black-Scholes pricing** Consider a call option on a stock with a local standard deviation of 42%/year. The option has 10 days to maturity (about 1/36 of a year), and is at-the-money in present value, that is, the present value of the strike price equals the stock price. The stock is selling for \$40 and the riskfree rate is 5%/year. No dividend will be paid during the next two weeks.

A. What are the variables T, S, B, and σ to be used in the option formula?

$T = 1/36$ (so $\sqrt{T} = 1/6$), $S = 40$, $B = 40$, and $\sigma = 0.42$

B. What is the call price from the approximation formula?

$$\begin{aligned} C &= \frac{S-B}{2} + 0.4\frac{S+B}{2}\sigma\sqrt{T} \\ &= 0 + 0.4 \times 40 \times 0.42 * 1/6 \\ &= 1.12 \end{aligned}$$

C. What is the corresponding European put price?

$S + P = B + C$

Since $B = S$, $P = C = 1.12$