FIN 550 Exam

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This is a closed-book examination. You may not use texts, notes, a crib sheet, calculator, cell phone, listening device, or any other electronics. Answer all questions as directed on the Blue Books provided. Be sure your answers are clearly marked. There are no trick questions on the exam. Good luck!

0. PLEDGE

The work on this exam is my own alone, and I have conformed with the rules of the exam and the code of the conduct of the Olin School.

Signed name ____

Printed name (write clearly) ____

1. True-False (25 points)

A. Unconstrained problems typically have interior solutions.

B. A local optimum of a linear program is a global optimum.

C. Absence of arbitrage is an unusual feature for option pricing models.

D. The product of the eigenvalues equals the determinant of a matrix.

E. A linear program with an unbounded dual always has an optimal solution.

2. Linear Programming (25 points)

Consider the following linear program:

Choose nonnegative x_1, x_2, and x_3 to
maximize $2x_1 + 2x_2 + 3x_3$, subject to
$x_1 + 2x_2 + 3x_3 \leq 6$ and
$x_1 + x_2 \leq 3$

An LP function in a program you have not used before gives the following solution to the LP.

choice variables: $x^* = (3, 0, 1)$

value of the program: 9

Lagrange multipliers of the constraints: $\lambda^* = (1, 1)$

Lagrange multipliers of positivity constraints: $\gamma^* = (0, 1, 0)$

A. Show that x^* is feasible.

B. Show that x^* is optimal.

C. Confirm the value of the program.

D. Use the Lagrange multipliers to approximate the new value of the program if we change 6 to 7 in the first constraint.

3. REGIME-SWITCHING (25 points) Consider a two-state Markov Chain in continuous time. Regime switches take place at the following rates:

state 1 $\rightarrow$ state 2	probability 0.05/year
state 2 $\rightarrow$ state 1	probability 0.05/year

Initially (at time $t = 0$), we are in state 1.

a. What is the matrix A in the ODE

$$\pi'(t) = A\pi(t)$$

describing the dynamics of the vector $\pi(t)$ of future regime probabilities?

b. Solve for the eigenvalues of A.

c. Solve for the associated eigenvectors.

d. Write down the general solution of the ODE.

e. Write down the particular solution corresponding to the initial condition that we start in state 1.

f. A project costing \$110,000 has a cash flow of \$12,000/year in state 1 and \$6,000/year in state 2. The cash flows continue forever. If the continuously-compounded interest rate is 10%/year, does this project have a positive NPV?

4. Kuhn-Tucker Conditions (25 points)

Consider the following optimization problem:

Choose c_u and c_d to
maximize $\frac{1}{2}(20c_u - c_u^2) + \frac{1}{2}(20c_d - c_d^2)$, subject to
$\frac{4}{5}(\frac{1}{4}c_u + \frac{3}{4}c_d) = 6$.

This is a single-period choice of investment for consumption in a binomial model with quadratic utility, initial wealth of 6, actual probabilities 1/2 and 1/2, risk-neutral probabilities 1/4 and 3/4, and riskfree rate of 25% (and therefore discount factor 4/5).

A. What are the objective function, choice variables, and constraint?

B. What are the Kuhn-Tucker conditions?

C. If we add constraints $c_u \geq 6$ and $c_d \geq 6$, what are the Kuhn-Tucker conditions now?

5. Bonus question (30 bonus points)

A. Solve the optimization problem in problem 4 without the extra constraints in part 4C.

B. Solve the optimization problem in problem 4 with the extra constraints in part 4C.

Useful formulas

For the problem

Choose $x \in \Re^N$ to
maximize $f(x)$
subject to $(\forall i \in \mathcal{E}) g_i(x) = 0$, and
$(\forall i \in \mathcal{I}) g_i(x) \leq 0$,

the Kuhn-Tucker conditions are

$\nabla f(x^*) = \sum_{i \in \mathcal{E} \bigcup \mathcal{I}} \lambda_i \nabla g_i(x^*)$
$(\forall i \in \mathcal{I}) \lambda_i \geq 0$
$\lambda_i g_i(x^*) = 0.$

For the LP

Choose $x \geq 0$ to
minimize $c^\top x$
subject to $Ax \geq b$,

the dual LP is

Choose $y \geq 0$ to
maximize $b^\top y$
subject to $A^\top y \leq c$.