FIN 550 Exam

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This is a closed-book examination. You may not use texts, notes, a crib sheet, calculator, cell phone, listening device, or any other electronics. Answer all questions as directed on the Blue Books provided. Be sure your answers are clearly marked. There are no trick questions on the exam. Good luck!

0. PLEDGE

The work on this exam is my own alone, and I have conformed with the rules of the exam and the code of conduct of the Olin School.

Signed name ____

Printed name (write clearly) ____

1. True-False (25 points)

A. Every unconstrained problem has at least one interior solution.

B. In a constrained problem with a concave objective function, a local maximum is always a global maximum.

C. Skewness of a random variable is always positive.

D. The sum of the eigenvalues of a negative semidefinite matrix is always negative.

E. A linear program with a bounded primal always has an optimal solution.

2. Linear Programming (25 points) Consider the following linear program:

Choose nonnegative x_1, x_2, x_3, and x_4 to
maximize $x_1 + 6x_2 + 4x_3 + 4x_4$, subject to
$x_1 + 3x_2 + 3x_3 + 5x_4 \leq 6$ and
$x_1 + x_2 \leq 3$

A. What is the dual linear program?

B. Solve the dual problem.

C. Use the solution to the dual problem to solve the primal problem.

D. Show that the strong duality theorem holds in this example.

3. Kuhn-Tucker Conditions (25 points)

Consider the following optimization problem:

Choose c_u and c_d to
maximize $\frac{2}{3}\log(c_u - 3) + \frac{1}{3}\log(c_d - 3)$, subject to
$\frac{4}{5}(\frac{1}{2}c_u + \frac{1}{2}c_d) = 6$,
$c_d \geq 6$,
and
$c_u \geq 6$.

This is a single-period choice of investment for consumption in a binomial model with translated log utility, initial wealth of 6, actual probabilities 2/3 and 1/3, risk-neutral probabilities 1/2 and 1/2, and riskfree rate of 25% (and therefore discount factor 4/5).

A. What are the objective function, choice variables, and constraints?

B. What are the Kuhn-Tucker conditions?

C. Solve the optimization problem.

4. REGIME-SWITCHING (25 points) Consider a two-state Markov Chain in continuous time. Regime switches take place at the following rates:

state 1 $\rightarrow$ state 2	probability 0.1/year
state 2 $\rightarrow$ state 1	probability 0.1/year

Initially (at time $t = 0$), we are in state 1.

A. What is the matrix A in the ODE

$$\pi'(t) = A\pi(t)$$

describing the dynamics of the vector $\pi(t)$ of future regime probabilities?

B. Solve for the eigenvalues of A.

C. Solve for the associated eigenvectors.

D. Write down the general solution of the ODE.

E. Write down the particular solution corresponding to the initial condition that we start in state 1.

F. A project costing \$45,000 has a cash flow of \$10,000/year in state 1 but loses money with cash flow $-$\$8,000/year in state 2. The cash flows continue forever. If the continuously-compounded interest rate is 10%/year, does this project have a positive expected NPV?

5. Bonus problem (30 bonus points)

Consider the Markov switching model in Problem 4 and in particular the project described in part 4E. Suppose that instead of continuing forever, the project can be costlessly (and irreversibly) abandoned anytime you want, without any cash flows at that time or subsequently. To get any credit for this problem, you must show your work, and there is no use just guessing.

A. What is the optimal strategy? You can restrict your analysis to the only sensible strategies (a) do nothing, (b) buy the project and never abandon it, and (c) buy the project and continue to operate it until state 2 occurs, at which time you abandon the project.

B. What is the NPV of the optimal strategy?

Useful formulas

For the problem

Choose $x \in \Re^N$ to
maximize $f(x)$
subject to $(\forall i \in \mathcal{E}) g_i(x) = 0$, and
$(\forall i \in \mathcal{I}) g_i(x) \leq 0$,

the Kuhn-Tucker conditions are

$\nabla f(x^*) = \sum_{i \in \mathcal{E} \bigcup \mathcal{I}} \lambda_i \nabla g_i(x^*)$
$(\forall i \in \mathcal{I}) \lambda_i \geq 0$
$\lambda_i g_i(x^*) = 0.$

For the LP

Choose $x \geq 0$ to
minimize $c^\top x$
subject to $Ax \geq b$,

the dual LP is

Choose $y \geq 0$ to
maximize $b^\top y$
subject to $A^\top y \leq c$.