INVESTMENTS
Lecture 2: Measuring Performance

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- portfolio returns
- unitization
- statistical measures of performance
- the use of benchmark portfolios

Security Returns

Recall that the *rate of return* on a security is the natural extension of the notion of an interest rate to securities with randomness. The rate of return over some time period is the change in value divided by the value at the start of the period. For example, for a bond paying a coupon:

$$R_t = \frac{P_t + C_t - P_{t-1}}{P_{t-1}} = \left(\frac{P_t}{P_{t-1}} - 1\right) + \frac{C_t}{P_{t-1}}$$

In the last expression, the first term is called the *capital gain (loss)*, and the second term is called the *income*. The rate of return is often stated in percent (also called points), or in basis points (100 basis points $=$ 1 percent). The obvious adjustment is made for stock splits or other distributions.

notation: R-return, P-price=quote less accrued interest, C-coupon, t-time

Simple portfolio returns

To compute the return for a portfolio, we follow the same basic approach of looking at the percentage change in value. Absent expenses or fees, inflows, and outflows, the portfolio return is given by

$$\begin{aligned} R_t &= \frac{\Sigma_i \, S_{i,t-1}(P_{i,t} + D_{i,t} - P_{i,t-1})}{\Sigma_i \, S_{i,t-1} P_{i,t-1}} \\ &= \sum_i \left(\frac{S_{i,t-1} P_{i,t-1}}{\Sigma_j \, S_{j,t-1} P_{j,t-1}} \right) \left(\frac{P_{i,t} + D_{i,t} - P_{i,t-1}}{P_{i,t-1}} \right) \\ &= \sum_i f_{i,t-1} R_{it} \end{aligned}$$

notation: R-return, S-#shares, P-price, D-dividend, f-fraction of wealth,
i (or j)-which security, t-time

In-class exercise: Rates of Return

Over the last day, Hamburger Corp stock has gone from \$50 to \$49 while going X-dividend with a dividend in the amount of \$3.00/share, and Pickel Corp stock has gone from \$75 to \$36 while splitting 2-for-1. What is the rate of return on each? What is the rate of return on a portfolio composed of 1000 shares of Hamburger Corp and 400 shares of Pickel Corp?

Some Adjustments to the Rate of Return

- Brokerage fees
- Fee income from securities lending
- Dividends and coupons
- Share dividends or rights offerings
- Proceeds from tendering shares
- Cost of exercising warrants, rights issues, or options
- variation for futures
- cash inflows or outflows

Unitization

When there is an inflow of cash to the portfolio, it would be unfair (and inaccurate) to credit that cash inflow to the manager's performance. It would be equally unfair to attribute cash outflows to bad performance by the manager. To neutralize the effect of cash inflows or outflows on the portfolio's value, we *unitize* the portfolio by thinking of the portfolio as being composed of units. Cash inflows and outflows are thought of purchases or sales of units, and do not affect the value per unit.

Unitization: example

For example, suppose we are computing the quarterly return to a pension portfolio worth \$10 million at the start of the quarter. One month into the quarter, the portfolio has grown to \$12 million. At that time, new contributions of \$6 million are added, making the whole portfolio worth \$18 million. Between then and the end of the quarter, the portfolio shrinks to \$16.2 million.

To compute the unitized return, think of the initial portfolio as consisting of, say, 5 units worth \$2 million each. A fictional unit worth \$2 million at the start would be worth \$12 million/5 = \$2.4 million a month in. The \$6 million cash injection buys 2.5 more units, so that after the cash injection the portfolio consists of 7.5 units worth \$2.4 million each. At the end of the quarter, each unit is worth \$16.2 million/7.5 = \$2.16 million. The unitized return on the portfolio was therefore (2.16-2)/2 = 8%, which is a gain of 800 basis points.

Unitization: a simplified view

Now that we have seen the thinking underlying unitization, let us consider a simplified way of doing the calculation. This simplified view is based on the observation that in between cash inflows and outflows, a unit has the same return as the portfolio. That means that the we can compute the return over the whole period using ordinary compounding. In the example in the previous slide, the portfolio rate of return is 20% over the first month and -10% over the remaining two months, for an overall rate of return of $(1+20\%)(1-10\%)-1 = 1.2*0.9-1 = 1.08-1 = 8\%$.

With many subperiods, we use the usual compounding rule that the total return (defined as one plus the rate of return) multiplies. For example, if the rates of return in four subperiods between inflows/outflows are R_a, R_b, R_c, and R_d, then the unitized rate of return for the whole period is

$$(1+R_a)(1+R_b)(1+R_c)(1+R_d)-1.$$

In-class Exercise: unitization

Use the following table of information to compute the unitized return for SuperGro fund for the first quarter. All additions to the fund occur at month-end.

month	starting value	ending value		net influx
		before net influx	after net influx	(outflux)
Jan	\$50 million	\$55 million	\$60 million	\$5 million
Feb	\$60 million	\$54 million	\$50 million	(\$4 million)
Mar	\$50 million	\$50 million	\$56 million	\$6 million

Unitization: Further Notes

As should be clear from the simplified formula, the unitized return is the same whatever the size of the initial unit. Changing the size of the initial unit changes the value of the unit at each later date proportionately. The constant of proportionality cancels when computing the return over the whole period.

Ideally, unitization makes performance completely comparable across managers with different patterns of cash inflows and outflows. However, transaction costs are usually higher in funds with large inflows and outflows, and therefore the comparison will not be entirely fair. In the presence of taxes, consistently receiving cash inflows can be an advantage since the inflows make it easy to maintain diversification without creating current tax liabilities. (If we take taxes seriously, performance measurement is difficult even without inflows and outflows, since it is difficult to know how a performance measure should account for unrealized gains and losses in the portfolio.)

Statistical Measures of Performance

Here are two commonly-used measures of performance that are motivated by the CAPM. In these definitions excess return means return less the riskless rate.

- Sharpe ratio:

$$\frac{\text{mean portfolio excess return}}{\text{standard deviation of portfolio excess return}}$$

- Jensen's alpha:

$$\text{mean portfolio excess return} - \beta \times (\text{mean market excess return})$$

where beta, the measure of market risk exposure, is

$$\beta = \frac{\text{covariance of the portfolio excess return and the market excess return}}{\text{variance of the market return}}$$

which is the slope of the regression of the portfolio excess return on market excess return and a constant.

Intuitions for the Sharpe ratio

Both the Sharpe ratio and Jensen's alpha are intended to put on a common basis portfolios that may have different risk exposures. In each case we subtract off the riskless rate (and use excess returns) because we want to reward managers based on relative performance.

The Sharpe ratio is appropriate for an entire portfolio (not just a subportfolio). With leverage, two portfolios with the same Sharpe ratio can give an equivalent distribution of results. For example, consider one portfolio with a mean excess return of 10% with a standard deviation of 40%, and compare it to a second portfolio with a mean excess return of 5% and a standard deviation of 20%. Then the second portfolio has the same mean and standard deviation as putting half of your money in the riskless asset and half in the first portfolio. This does not tell us whether we might be even better off combining the two portfolios, since that depends on the portfolios covariances which is not an input to the Sharpe measure.

Intuition for Jensen's alpha

The Jensen measure is intended to measure the incremental performance for an addition to a large well-diversified portfolio that mimics the market. We can associate with the member the profitability of a net arbitrage position that goes long the managed portfolio and goes short the riskless asset and the market in proportions that neutralize market risk. For example, consider a managed portfolio with a beta of $1/2$. Then Jensen's alpha measures the average profit of investing \$1 in the managed portfolio, obtaining the funds from borrowing \$.50 (shorting the riskless asset) and shorting \$.50 worth of the market portfolio.

Jensen's alpha can also be interpreted as the deviation from the SML in the CAPM. Recall that the SML tells us how much expected return we should give for a certain beta in the CAPM. Deviations from the SML are interpreted to be the result of superior performance but of course they can be due to luck instead.

In-class exercise: Sharpe ratio and Jensen's alpha

The following table gives statistics describing returns of your portfolio and the market index during one year. Compute the Sharpe measure and Jensen's alpha for your portfolio. All numbers are monthly. The average riskless rate was 0.005 and the covariance between the market and your portfolio was 0.0015.

	mean	variance	std dev
yours	0.00	0.0100	0.10
S&P 500	-0.01	0.0025	0.05

Sharpe ratio and Jensen's alpha: theoretical considerations

The original motivations for the Sharpe measure and Jensen's alpha were heuristic. Subsequent analysis (as in my papers with Ross in the June, 1985 issue of the *Journal of Finance*) suggests they may be useful for evaluating stock picking (if we have enough time periods to provide statistical significance) but not for evaluating market timing.

Performance Measurement

Evaluating the performance of a portfolio manager is difficult because the large amount of randomness in portfolio returns makes it difficult to distinguish luck from skill. A typical stock portfolio may have a standard deviation of 25 to 30 percent per year. Even very significant performance (say 5% per year) can be distinguished from luck only after a long period.

Financial economists have developed a number of tools for measuring performance that attempt to get around this problem. Unfortunately, they are not very reliable. The industry response to the problem has been to restrict significantly what portfolio managers can choose, and further to measure performance relative some benchmark portfolio the manager is supposed to emulate. Measuring performance compared to a nearby benchmark is much easier than measuring performance of an arbitrary portfolio. Using a benchmark, together with placing constraints on the portfolio, is also intended to ensure the portfolio choice is not too crazy.

Benchmark Portfolios

It is common for the money manager and the plan sponsor to agree in advance on a benchmark to be used for judging performance. Normally, the benchmark is chosen to be a non-discretionary version of the same portfolio. For example the benchmark for a portfolio of large stocks might be the S&P 500, a benchmark for a portfolio of low-cap firms might be the Wiltshire 500, and the benchmark for a portfolio of Treasury Bills might be the 3-month Treasury.

Various comparisons can be done between the managed portfolio and the benchmark. The simplest comparison looks at the net excess of the return on the managed portfolio over the benchmark. When this difference is positive, we interpret this as superior performance. When this difference is negative, we interpret this as inferior performance.

Benchmark Definitions

- Equal-weighted portfolio: assumes that equal proportions of wealth are invested in each asset
- Value-weighted portfolio: assumes that market proportions of supply are mirrored in the portfolio
- Rebalancing frequency: says how often the portfolio is changed to reflect market conditions. For an equal-weighted portfolio, it usually makes sense to rebalance the portfolio infrequently even if we compute returns daily.

How easy is it to Beat the Benchmark?

A benchmark portfolio is intended to represent the return to a passively managed portfolio. In fact, the benchmark portfolio is a paper portfolio and may not represent the performance of a feasible passive strategy. For example, the trades in the benchark portfolio may assume execution at some closing price, while actual trades would not execute so well due to the bid-ask spread. (Interestingly, this is a special problem for equally-weighted indices that are rebalanced daily, since an equally weighted strategy implicitly buys more of a stock whose closing price is artifically low.) Also, the benchmark portfolio usually assumes no brokerage fees.

There are also reasons why a benchmark portfolio might understate realistic performance. For example, quoted benchmarks often do not include dividend income in computing returns. Also, security lending fees (to accomodate investors desiring to take short positions) are not included in benchmarks.