

Problem Set 3: Linear Algebra
FIN 500J Mathematical Foundations for Finance
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Do all the calculations in the problem set by hand and show intermediate steps so the TAs will know you did not just copy the answers from the sheet with selected answers. It is okay to work together or get help from anyone else, provide you do the write-up and any computer work yourself.

The TAs will grade the homeworks.

1. Consider the matrices

$$B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

1. Compute the following.

A. B^T (the transpose of B)

B. BC (the product of B and C)

C. $\det(CC^T)$ (the determinant of C times the transpose of C)

D. $\det(C^TC)$ (the determinant of the transpose of C times C)

2. Consider the two equations

$$2x_1 - x_2 = 1$$

$$7x_1 - 2 = 3x_2$$

A. Write these equations in the form $Ax = b$. What are A and b ?

B. Compute A^{-1} .

C. Compute x .

3. Consider the matrix

$$D = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix}$$

A. Compute the eigenvalues λ_1 and λ_2 of D .

B. Compute corresponding eigenvectors.

C. Let $x_0 = (3, 2)^T$. Write x_0 as a linear combination of the eigenvectors.

D. Use the eigenvalues and eigenvectors to compute $D^5 x_0$.

4. Extra for Experts (an optional problem for students of superior ability, preparation, or ambition) Prove your answers.

Let

$$F = \begin{pmatrix} 3 & 7 \\ 2 & 8 \end{pmatrix} \text{ and } g = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

(a) Then, how many digits are there in the largest element of

$$y = F^{100000} g?$$

(b) What is the difference between the first element of y and the second element of y ?