

Problem Set 3, selected answers: Linear Algebra  
FIN 500J Mathematical Foundations for Finance  
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1. Consider the matrices

$$B = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

1. Compute the following.

A.  $B^T$  (the transpose of  $B$ )

B.  $BC$  (the product of  $B$  and  $C$ )

$$BC = \begin{pmatrix} 2 & 0 & 1 \\ -2 & 1 & -1 \end{pmatrix}.$$

C.  $\det(CC^T)$  (the determinant of  $C$  times the transpose of  $C$ )

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D.  $\det(C^TC)$  (the determinant of the transpose of  $C$  times  $C$ )

0

2. Consider the two equations

$$2x_1 - x_2 = 1$$

$$7x_1 - 2 = 3x_2$$

A. Write these equations in the form  $Ax = b$ . What are  $A$  and  $b$ ?

B. Compute  $A^{-1}$ .

$$A^{-1} = \begin{pmatrix} -3 & 1 \\ -7 & 2 \end{pmatrix}$$

(This could be  $-A^{-1}$ , depending on the answer to part A. above.)

C. Compute  $x$ .

$$x = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$$

3. Consider the matrix

$$D = \begin{pmatrix} 0 & 2 \\ -1 & -3 \end{pmatrix}$$

A. Compute the eigenvalues  $\lambda_1$  and  $\lambda_2$  of  $D$ .

One eigenvalue is  $-1$ .

B. Compute corresponding eigenvectors.

The corresponding eigenvector to the eigenvalue  $-1$  is any nonzero vector proportional to  $(-2, 1)^T$ .

C. Let  $x_0 = (3, 2)^T$ . Write  $x_0$  as a linear combination of the eigenvectors.

D. Use the eigenvalues and eigenvectors to compute  $D^5 x_0$ .

$$D^5 x_0 = \begin{pmatrix} 214 \\ -219 \end{pmatrix}$$