Problem Set 1: Probability
FIN 500J Mathematical Foundations for Finance
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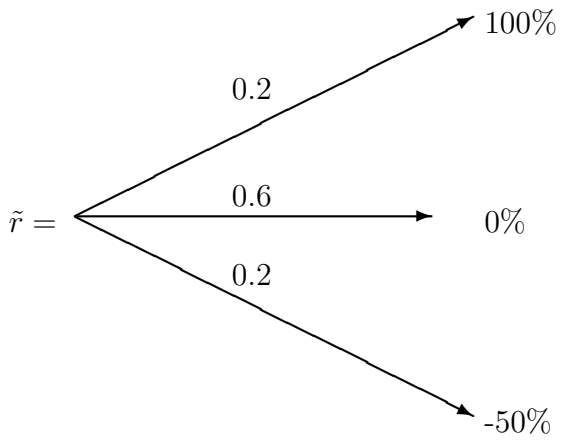
Here are some answers to help you see whether you are on the right track. The answers to questions 2 and 3 are less detailed than what you need to give in your write-ups.

1. Consider a model of stock returns using a trinomial model. The stock return in any period is 100% with probability 0.2, 0% with probability 0.6 and −50% with probability 0.2.



A. Compute the expected return $E[\tilde{r}]$.

$$E[\tilde{r}] = .2 \times 100 + .6 \times 0 + .2 \times (-.50) = .1$$

B. Compute the variance of return $\text{var}[\tilde{r}]$.

$$\begin{aligned} \text{var}[\tilde{r}] &= E[\tilde{r}^2] - (E[\tilde{r}])^2 \\ &= .2 \times 1 + .6 \times 0 + .2 \times .25 - .1^2 \\ &= .24 \end{aligned}$$

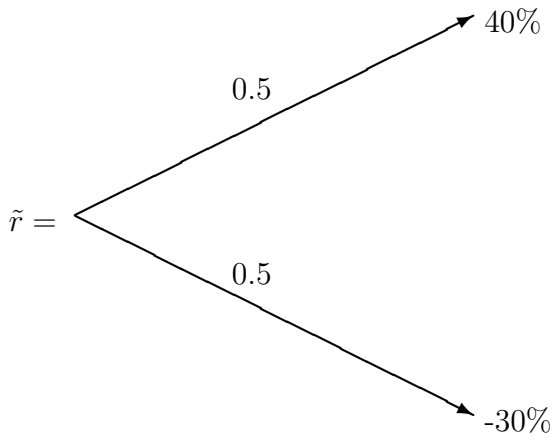
C. Compute the standard deviation of return $\text{std}[\tilde{r}]$

$$\text{std}(\tilde{r}) = \sqrt{.24} \approx .49$$

Note: these numbers can also be computed as percentages, in which case the mean is 10, the variance is 2400, and the standard deviation is $\sqrt{2400} \approx 49$.

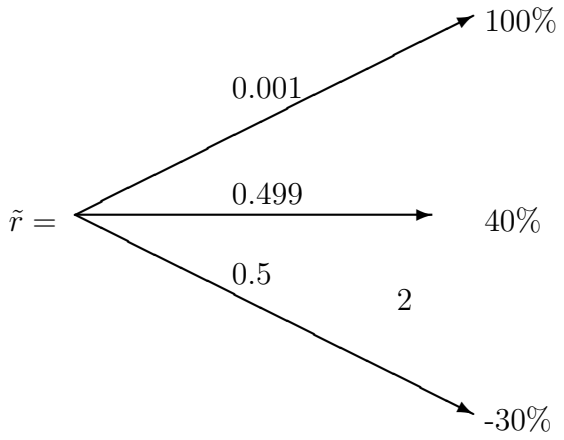
2. (skewness and kurtosis) Compute the skewness and kurtosis for the following three probability distributions:

A. The stock return is +40% with probability 0.5 and −30% with probability 0.5:



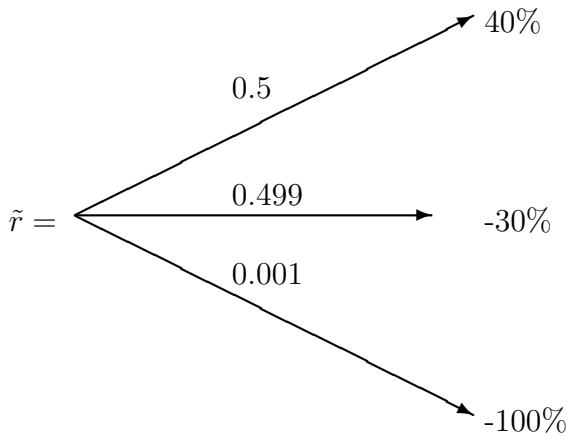
$$\text{skewness} = E[(x - \mu_x)^3]/{\sigma_x}^3 = 0$$
$$\text{kurtosis} = E[(x - \mu_x)^4]/{\sigma_x}^4 = 1$$

B. The stock return is +100% with probability 0.001, +40% with probability 0.499, and −30% with probability 0.5:



$$\text{skewness} = E[(x-\mu_x)^3]/{\sigma_x}^3 = 0.0137$$
$$\text{kurtosis} = E[(x-\mu_x)^4]/{\sigma_x}^4 = 1.0399$$

C. The stock return is +40% with probability 0.5, −30% with probability 0.499, and −100% with probability 0.001:



$$\text{skewness} = E[(x-\mu_x)^3]/{\sigma_x}^3 = -0.0197$$
$$\text{kurtosis} = E[(x-\mu_x)^4]/{\sigma_x}^4 = 1.0627$$

D. Interpret the change in skewness and the change in kurtosis in moving from case A to case B and from case A to case C.

3. Consider the following stock returns for three years:

year	ABC	XYZ
2000	60%	40%
2001	-40%	50%
2002	10%	-30%

A. Compute the sample means, variances, and covariance of the two stock returns.

$$\mu_{ABC} = .1, \mu_{XYZ} = .2$$
$$\text{var}(ABC) = .25, \text{var}(XYZ) = .19, \text{cov}(ABC, XYZ) = -.025$$

B. Compute the estimate of the regression coefficient of the return of ABC stock on the return of XYZ stock.

$$\beta = \frac{\text{cov}(ABC, XYZ)}{\text{var}(XYZ)} = -.1315789$$

C. If XYZ stock has a bad year next year, do you expect ABC stock to do well?

D. (extra for experts) Do you think holding shares of ABC can be used to hedge holding shares of XYZ?