Fin 500J Homework 2

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due on Aug. 24th, 2009

Problem 1.

(1)Let

$$f(x, y, z) = 2x + 3y^2 - sin(z),$$

compute the gradient of function f, ∇f.

(2)Given

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}, \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

and

$$y_1 = 6x_1 + 4x_2 - e^{x_3}, \quad y_2 = sin(x_1) + 2x_2 + 3x_3^2,$$

compute the partial derivative matrix $\frac{\partial y}{\partial x}$.

Problem 2. Determine the real root of

$$f(x) = 5x^3 - 5x^2 + 6x - 2 :$$

(1)Graphically (plot f(x) in [-1,2] in Matlab)

(2)Programming in Matlab using bisection to locate the root. Employ initial guesses of $a = 0$ and $b = 1$ and do 8 iterations. Print out your code, graph and results.

(3)Finding the real root using Matlab function 'fzero'.

Problem 3. Determine the lowest positive root of

$$f(x) = 8sin(x)e^{-x} - 1 :$$

(1)Graphically (plot f(x) in [0, 2] in Matlab)

(2)Programming in Matlab using the Newton-Raphson method (three iterations, $x_0 = 0.3$).

(3)Programming in Matlab using the secant method (three iterations, $x_0 = 0.5$ and $x_1 = 0.4$).

Print out your code, graph and results.

(4)Finding the lowest positive root using Matlab function 'fzero'.

Problem 4. For each of the following functions, find the critical points and classify these as local max, local min, saddle point or 'can't tell':

$$(1)xy^2 + x^3y - xy, \quad (2)x^2 + 6xy + y^2 - 3yz + 4z^2 + 6x + 17y - 2z.$$

Problem 5. A firm's production function is given by

$$Q = 2L^{1/2} + 3K^{1/2}$$

where Q, L and K denote the number of units of output, labor and capital. Labor costs are $2 per unit, capital costs are $1 per unit and output sells at $8 per unit. Find the maximum profit and the values of L and K at which it is achieved.