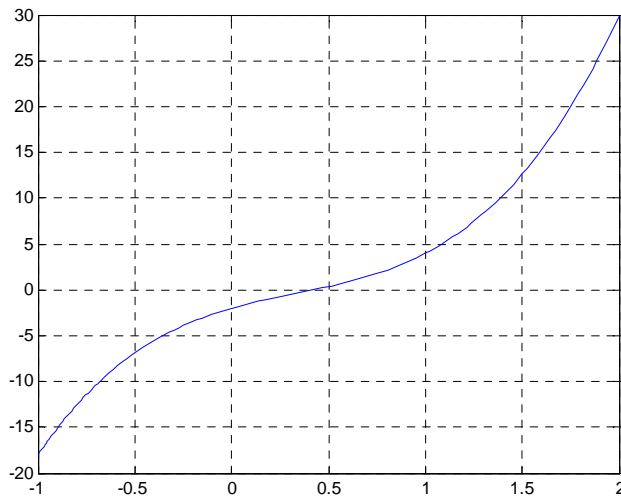Problem 1.

(1) $\nabla f = \begin{pmatrix} 2 \\ 6y \\ -\cos(z) \end{pmatrix}$

(2) $\dfrac{\partial y}{\partial x} = \begin{pmatrix} 6 & \cos(x) \\ 4 & 2 \\ -e^{x_3} & 6x_3 \end{pmatrix}$

Problem 2.

(1) As we can see from the graph below, the root of function f(x) is approximately 0.4.

(2) Matlab code:

```
a=0; b=1;
u=5*a^3-5*a^2+6*a-2; v=5*b^3-5*b^2+6*b-2;
for i=1:8
c=(a+b)/2
fc=5*c^3-5*c^2+6*c-2
if u*fc<0
b=c; v=fc;
else
a=c;u=fc;
end
end

Results:
c =0.5000
```

```
fc =0.3750
c =0.2500
fc =-0.7344
c =0.3750
fc =-0.1895
c =0.4375
fc =0.0867
c =0.4063
fc =-0.0525
c =0.4219
fc =0.0168
c =0.4141
fc =-0.0179
c =0.4180
fc =-5.8562e-004
```

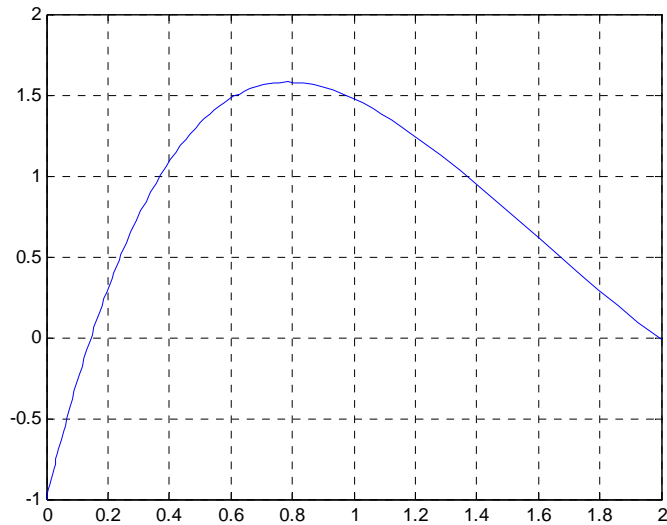So, if we do 8 iterations, the estimator of the root is about 0.4180.

(3) >> fzero(f,[0,1])
>> ans =0.4181

Problem 3.
   (1) From the graph, we can see that the root is approximately 0.1

(2) Matlab code:
   Firstly, we create two m file, FF.m and FFP.m (the name of these two functions and m files can be arbitrary, but make sure that they are the same)

```
% function FF, we save it as FF.m
function [FF]=FF(X)
FF=8*sin(X)*exp(-X)-1;


%function FFP, we save it as FFP.m
function [FFP]=FFP(X)
```

```
 FFP=-8*sin(X)*exp(-X)+8*cos(X)*exp(-X);

%Then, we do 3 iterations running the following program
X=0.3;
for i=1:3
X=X-FF(X)/FFP(X)
FF(X)
End
```

Results:

X =0.1450
ans =-5.0966e-011

X =0.1450
ans =-2.2204e-016

X =0.1450
ans =0

Therefore, the root is 0.1450 using the Newton-Raphson method.

(3) Matlab code:
Firstly, define function FSe and save it as FSe.m

```
function [FSe]=FSe(X)
FSe=8*sin(X)*exp(-X)-1;
```

Then, do 3 iterations (note that in this example, we need do more than 3
iterations to get a more accurate estimator using Secant method)


```
X=zeros(5,1);
X(1)=0.5; X(2)=0.4;
for i=3:5;
X(i)=X(i-1)-FSe(X(i-1))*(X(i-1)-X(i-2))/(FSe(X(i-1))-FSe(X(i-2)));
end
for i=1:5
 Xi=X(i)
 FXi=FSe(X(i))
 End
```

Results:
Xi =0.5000
FXi =1.3263

Xi =0.4000
FXi =1.0883

Xi =-0.0572
FXi =-1.4846

Xi =0.2066
FXi =0.3347

```
 Xi =0.1581
 FXi =0.0751

 After three iterations, the estimator of the root is 0.1581 (you can do
 more iteration to get a better estimator.)

(4) >> fzero(f, 0.4)
    ans =0.1450
```