# Fin 500J Homework 3

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Problem 1. Consider the problem of maximizing $f(x, y, z) = xyz + z$, subject to the constraints $x^2 + y^2 + z \leq 6, x \geq 0, y \geq 0, z \geq 0$.

(1) Write out a complete set of first order conditions for this problem.

(2) Determine whether or not the constraint $x^2+y^2+z \leq 6$ is binding at any solution.

(3) Find a solution of the first order conditions that includes $x = 0$.

(4) Find three equations in the three unknowns $x, y, z$ that must be satisfied if $x \neq 0$ at the solution.

(5) Show that $x = 1, y = 1, z = 4$ satisfies these equations.

(6) Write out the bordered Hessian and check the second order conditions for the solution in (5).

(7) Use the sensitivity analysis to estimate the maximum value of $f(x, y, z)$ on the constraint set $x^2 + y^2 + z \leq 6.2, x \geq 0, y \geq 0$ and $z \geq 0$.