

Fin 500J Homework 6

Yajun Wang

Olin Business School

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Problem 1. (i) Suppose that the *p.d.f.* of a certain random variable X has the following form:

$$f(x) = \begin{cases} cx & 0 < x < 4 \\ 0 & \text{otherwise,} \end{cases}$$

where c is a given constant. Determine the value of c and also the values of $Pr(1 \leq X \leq 2)$ and $Pr(X > 2)$.

(ii) Suppose that a random variable X has a uniform distribution on the interval $[-2, 8]$, find (i) the *p.d.f.* of X ; (ii) the value of $Pr(0 < X < 7)$; (iii) the mean and the variance of X .

Problem 2 Suppose that the joint *p.d.f.* of two random variables X and Y is as follows:

$$f(x, y) = \begin{cases} c(x^2 + y) & 0 \leq y \leq 1 - x^2 \\ 0 & \text{otherwise.} \end{cases}$$

Determine (i) the value of the constant c ; (ii) $Pr(0 \leq X \leq \frac{1}{2})$; (iii) $Pr(Y \leq X + 1)$.

Problem 3 (i) Suppose that X and Y are independent Poisson random variables such that $Var(X) + Var(Y) = 5$. Evaluate $P(X + Y < 2)$.

(ii) Suppose that X_1 and X_2 are independent random variables, and X_i has an exponential distribution with parameter $\beta_i (i = 1, 2)$. Find $Pr(X_1 > kX_2)$, where $k > 0$ is a constant.

Problem 4 Let X and Y have a continuous distribution with joint *p.d.f.*

$$f(x, y) = \begin{cases} x + y & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Compute the covariance $Cov(X, Y)$.