Fin 524 Final Exam Answers, October 18, 2007

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This is a closed-book examination. Answer all questions as directed. Mark your answers directly on the examination. On the valuation question, make sure your answer is clearly indicated. There are no trick questions on the exam. Good luck!

A. Binomial Option Pricing in One Period *40 points*

Riskless bond (interest rate is 25%):

$$100 \longrightarrow 125$$

Stock:

$$100 \begin{array}{l} \nearrow 245 \\ \searrow 45 \end{array}$$

European put with a strike price of 120:

$$? \begin{array}{l} \nearrow ? \\ \searrow ? \end{array}$$

1. What are the payoffs of the European put in the up and down states?

$$\begin{array}{l} \nearrow 0 \leftarrow \text{up state} \\ \searrow 75 \leftarrow \text{down state} \end{array}$$

2. What are the risk-neutral probabilities for the two states tomorrow?

$r = 1.25$
$u = 245/100 = 2.45$
$d = 45/100 = 0.45$

$$\begin{aligned} \pi_u &= \frac{r-d}{u-d} \\ &= \frac{1.25-0.45}{2.45-0.45} \\ &= \frac{0.8}{2} \\ &= 0.4 \end{aligned}$$

$\pi_d = 1 - \pi_u = 0.6$

3. What is the no-arbitrage price of the European put today?

$\frac{1}{1.25}(0.4 \times 0 + 0.6 \times 75) = 36$

4. What is the portfolio of stocks and bonds that replicates the put?

$$\begin{array}{rcr} 245n_s + 1.25n_b & = & 0 \\ 45n_s + 1.25n_b & = & 75 \\ \hline 200n_s & = & -75 \end{array}$$

$n_s = -3/8$
$n_b = \frac{-245n_s}{1.25} = 73.5$

check: $-\frac{3}{8} \times 100 + 73.5 = -37.5 + 73.5 = 36$

B. Concepts (true or false) *20 points*

1. To convert a good deal into a sure thing, we can try to hedge using derivative securities and match cash flows using borrowing and lending.

true

2. The Black-Scholes model prices American put and call options.

false (Black-Scholes prices European options)

3. American call options on stocks that pay no dividends are often worth more than corresponding European call options.

false (given no dividends, exercising an American call early is dominated by waiting to maturity)

4. There must be an arbitrage if the 3-month futures price of corn is less than the spot price.

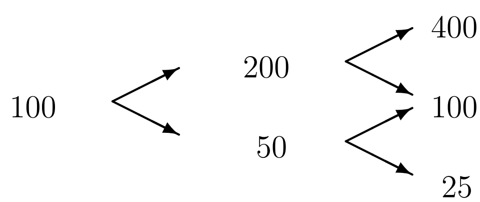
false (it may not be possible to short)

5. Orange County lost a lot of money trading commodity futures options.

false

C. Binomial Stock Option Pricing *40 points*

The short riskless interest rate is 25% per period. The stock price now is \$100 and will double or fall by half in each period:



The actual probability of an increase is 60% and the actual probability of a decrease is 40%. Consider European and American options with a strike price of \$75 and maturity two periods from now.

1. What are the risk-neutral probabilities?

$r = 1.25$
$u = 2$

$d = 1/2$

$\pi_u = \frac{1.25-0.5}{2.0-0.5} = \frac{0.75}{1.5} = \frac{1}{2}$

2. What are the European put values at each node?

$$\begin{tikzcd} & & 0 \\ & 0 \arrow[ur] \arrow[dr] & \\ 8 \arrow[ur] \arrow[dr] & & 0 \\ & 20 \arrow[ur] \arrow[dr] & \\ & & 50 \end{tikzcd}$$

$\frac{1}{1.25}\left(\frac{1}{2}0 + \frac{1}{2}50\right) = 20$
$\frac{1}{1.25}\left(\frac{1}{2}0 + \frac{1}{2}20\right) = 8$

3. What are the American put values at each node?

$$\begin{tikzcd} & & 0 \\ & 0 \arrow[ur] \arrow[dr] & \\ 10 \arrow[ur] \arrow[dr] & & 0 \\ & 25 \arrow[ur] \arrow[dr] & \\ & & 50 \end{tikzcd}$$

$\max(0, 75 - 200) = \max(0, -125) = 0$
$\max\left(\frac{1}{1.25}\left(\frac{1}{2}0 + \frac{1}{2}50\right), 75 - 50\right) = \max(20, 25) = 25$
$\max\left(\frac{1}{1.25}\left(\frac{1}{2}0 + \frac{1}{2}25\right), 75 - 100\right) = \max(10, -25) = 10$

4. What is the price of a European call option with strike \$75 maturing two periods from now? Use put-call parity.

$B + C = S + P$

$$\begin{aligned} C &= S + P - B \\ &= 100 + 8 - \frac{1}{1.25^2}75 \\ &= 100 + 8 - 48 \\ &= 60 \end{aligned}$$

D. Bonus question *20 points*

Let S_0 be the initial price of a share of stock that pays no dividends. Let r be one plus the riskfree rate, let μ be one plus the mean stock return, and let σ be the standard deviation of the stock return, all per period. What is the price today of a claim maturing 10 periods from now that pays the average stock price over the next 10 periods?

$$\begin{aligned}
price &= \frac{1}{r^{10}} E^*[\frac{S_1 + S_2 + ... + S_{10}}{10}] \\
&= \frac{1}{10r^{10}} \left(E^*[S_1] + E^*[S_2] + ... + E^*[S_{10}]\right) \\
&= \frac{1}{10r^{10}} (S_0 r + S_0 r^2 + ... + S_0 r^{10}) \\
&= \frac{S_0}{10r^{10}} (r + r^2 + ... + r^{10})
\end{aligned}$$

Note: $E^*[S_t] = r^t$ because in the risk-neutral probabilities, the stock price grows at the riskfree rate.

There are many other ways of expressing the formula, for example using the geometric formula.