Practice problems for Lecture 2. Answers.

1. A Simple Option Pricing Problem in One Period

Riskless bond (interest rate is 50%):

100 $\longrightarrow$ 150

Stock:

50 $\begin{cases} 125 \\ 50 \end{cases}$

Derivative security (call option with a strike of 80):

? $\begin{cases} 45 \\ 0 \end{cases}$

a. What is the portfolio of the stock and the bond that replicates the option?

Suppose the replicating portfolio has α_S dollars worth of stock and α_B dollars worth of bonds, replicating in both states implies that:

$$(125/50)\alpha_S + (150/100)\alpha_B = 45 \quad \text{(up state)}$$

and

$$(50/50)\alpha_S + (150/100)\alpha_B = 0 \quad \text{(down state)}.$$

Solving for α_S and α_B, we have that $1.5\alpha_S = 45$ or $\alpha_S = 30$ (which is purchase of 3/5 of a share) and $30 + 1.5\alpha_B = 0$ or $\alpha_B = -20$ (which shorting 1/5 of a bond).

b. What is the price of the portfolio?

The price of the replicating portfolio is $\alpha_S + \alpha_B = 30 - 20 = 10$.

c. What are the risk-neutral probabilities of the two states? Verify that your answer gives the correct price for the stock, the bond, and the option. Reminder: the formula for the risk-neutral probability of an up move is $\frac{r-d}{u-d}$.

$r = \frac{150}{100} = 1.5$, $u = \frac{125}{50} = 2.5$, $d = \frac{50}{50} = 1$

The risk-neutral probability of the up state is $(r-d)/(u-d) = (1.5-1)/(2.5-1) = 1/3$. The risk-neutral probability of the down state is $1 - 1/3 = 2/3$. For the stock, the price is 50 as is given using risk-neutral probabilities:

$$
\begin{aligned}
\text{Price} \;&=\; \frac{1}{1.5}\left(\frac{1}{3}125 + \frac{2}{3}50\right) \\
&=\; \frac{2}{3}\left(\frac{125}{3} + \frac{100}{3}\right) \\
&=\; 50.
\end{aligned}
$$

d. (thought question - answer in a sentence or two of ordinary length) Suppose we have inherited many shares of the stock and would like to trade them to diversify but we are precluded from trading the stock due to terms of an inheritence. If the inheritence does not preclude trading the option, how can this help us? (Note: do not perform any computations in this answer.)

If we short a portfolio of options and bonds that replicates the stock, we can undo the risk exposure without violating at least the letter of the terms of our inheritence.

2. General Concepts (very short answers)

a. We buy cheap gold futures in one market and sell the identical contract for more in another market to lock in a riskless profit. This is most related to which concept: arbitrage, dominance, hedging, or value-at-risk?

arbitrage

b. If the call option price equals the underlying stock price, we would rather buy the stock than the option because the stock is worth more in most contingencies and is never worth less. This is most related to which concept: arbitrage, dominance, hedging, or value-at-risk?

dominance

c. We sell short a portfolio of the stock and the bond that has the same terminal payoff as employee stock options we own. This is best described by which concept: arbitrage, dominance, hedging, or value-at-risk?
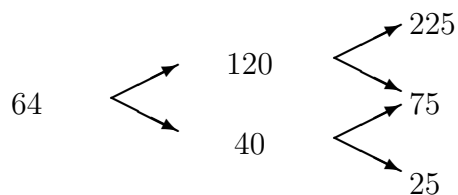
hedging

d. Name three types of options on common stock.

call, put, digital (many other possible answers)
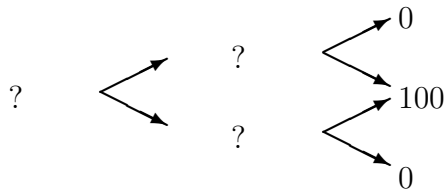
3. A Simple Option Pricing Problem in Two Periods

Riskless bond (riskfree interest rate is 25%)

16 $\longrightarrow$ 20 $\longrightarrow$ 25

Stock price:

64 $\Big\langle$ 120 $\Big\langle$ 225 / 75

40 $\Big\langle$ 75 / 25

Derivative security (European binary/digital option paying 100 when the stock price is between 50 and 100 and paying zero otherwise):

The actual probabilities are $1/3$ for up and $2/3$ for down at each node.

a. What are the risk-neutral probabilities of up and down? (The risk-neutral probabilities are the same at every node: compute them at the first node. Reminder: the formula for the risk-neutral probability of an up move is $(r-d)/(u-d)$. Advice: check to make sure the stock price is consistent with your answer.)

$r = 20/16 = 5/4$, $u = 120/64 = 15/8$, $d = 40/64 = 5/8$
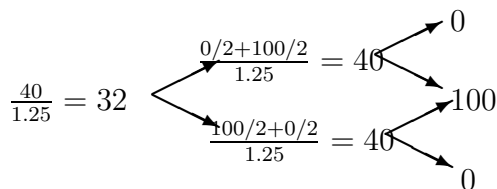
The risk-neutral probability of up is

$$(r-d)/(u-d) = (10/8 - 5/8)/(15/8 - 5/8) = 1/2$$

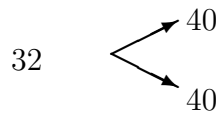and therefore the risk-neutral probability of down is $1 - 1/2 = 1/2$.

Stock pricing check:

$$\frac{120/2 + 40/2}{1.25} = \frac{80}{1.25} = 64$$

b. What is the value of the binary option at each node?

c. In the portfolio strategy that replicates the binary option, what are the holdings in the stock and the bond at the initial node?

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$$32 \begin{cases} \nearrow 40 \\ \searrow 40 \end{cases}$$

This is riskless, so just invest \$32 in the bond and 0 in the stock.

d. (thought question – answer in a sentence or two of ordinary length) Suppose you have a view that volatility will be smaller than most people expect. Your investment banker has proposed the purchase of these binary options as a way of profiting if your view is correct. Discuss.

This is a good way of betting that the stock will not go up or down too much, but only if your investment banker does not charge too much for the option!