Practice problems for Lecture 4.

1. Black-Scholes option pricing

Suppose the stock price is 40 and we need to price a call option with a strike of 45 maturing in 4 months. The stock is not expected to pay dividends. The continuously-compounded riskfree rate is 3%/year, the mean return on the stock is 7%/year, and the standard deviation of the stock return is 40%/year. (The Black-Scholes formula is given at the end of the homework.)

a. What are S and B?

b. What are x_1 and x_2?

c. $N(x_1) = 0.3627026$ and $N(x_2) = 0.2802213$ (confirm for yourself if you like). What is the Black-Scholes call price?

d. What is the Black-Scholes price for the European put with the same strike and maturity?

e. Conceptual question: Since the put option is worth more alive than if exercised now ($45 - 40 = 5 < 6.57586$), can we conclude that an American version of the put is worth the same as the European put?

2. Approximation

As noted in class, for near-the-money call options, a good approximation to the option price is

$$\frac{S-B}{2} + .4\frac{S+B}{2}\sigma\sqrt{T}$$

where S is the stock price, B the present value of receiving the strike at maturity, σ is the local standard deviation, and T is the time to maturity.

Consider an at-the-money call option that is one week to maturity on a stock with a local standard deviation of 35%/year. If the stock is selling for \$50 and the continuously-compounded riskfree rate is 1%/year, then the Black-Scholes call option price is \$0.9727852.

a. What is the call price from the approximate formula?

b. What is the error from using the approximate price?

c. What is the corresponding European put price using the approximation? (Use put-call parity. The Black-Scholes put price is \$0.963.)

Useful Formula

The Black-Scholes call price is

$$C(S,T) = SN(x_1) - BN(x_2),$$

where S is the stock price, $N(\cdot)$ is the cumulative normal distribution function, T is time-to-maturity, B is the bond price $Xe^{-r_f T}$, r_f is the continuously-compounded riskfree rate, σ is the standard deviation of stock returns,

$$x_1 = \frac{\log(S/B)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T},$$

and

$$x_2 = \frac{\log(S/B)}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T}.$$

Note that $\log(\cdot)$ is the natural logorithm.