OPTIONS and FUTURES
Lecture 5: Forwards, Futures, and Futures Options

Philip H. Dybvig
Washington University in Saint Louis

- Spot (cash) market
- Forward contract
- Futures contract
- Options on futures

Copyright © Philip H. Dybvig 2004

Commodity spot market (cash) prices

- Two-way arb in stock indices, gold, and silver
- One-way arb in many storable commodities
  - can go long physical commodity
  - cannot go short
- Adjust return by:
  - storage cost
  - use value
- $E^*[Return] \leq riskfree\ rate - use\ value + storage\ cost$
- No arb or weak arb available for nonstorable goods (e.g. electricity)

Dependence of futures price on maturity

- Futures price = risk-adjusted expected spot rate
- If storable and the stock is ample, cannot jump up or down
  - gold and silver
- If storable with stockouts, can jump down
  - agriculturals: at expected harvest
  - oil: at anticipated easing of supply
- If not storable (except at high cost), can jump up or down
  - electricity and natural gas
  - oil, if at storage capacity
- paper (interesting mkt, futures but no spot)

Forward contracts

A *forward contract* is an agreement to buy a specified quantity of some good for a pre-arranged price at a pre-arranged date in the future. For example, a jewelry manufacturer that uses silver might agree to buy 5,000 troy ounces of silver six months from now for $\$640$ per troy ounce. Some properties of forward contracts:

- no money or goods change hands before maturity
- good for hedging or speculating
- can buy and sell risk without holding the physical
- price discovery as in other security markets

In-class exercise: forward contracts

One year ago you contracted to buy forward 5,000 bushels of wheat for delivery today at a price of 300 cents per bushel. Today the spot price for wheat is 330 cents per bushel. Assuming the wheat delivered on the forward contract is sold in the spot market, what is the cash flow?

One year ago you entered a two-year forward contract (for delivery one year from now) to buy 100 troy ounces of gold for $440 per ounce. Today the one-year forward price is $462 per ounce. What is the value of the position today? (Assume a riskless rate of 10 percent per year.) Hint: set up an arb to move all the value to the present.

The forward price of an ideal storable commodity

Suppose we have a forward contract on one share of common stock that pays no dividends. Then we are indifferent about receiving the stock today and receiving it at the maturity of the contract. Also, paying the forward price F at the maturity is equivalent to paying Fe^{-r_fT} today, where r is the interest rate and T is the time to maturity of the forward contract. Therefore, the forward transaction is equivalent to buying a share of stock today for Fe^{-r_fT}. In the absence of arbitrage, we have that this is today's stock price, that is, $Fe^{-r_fT} = S$ or $F = Se^{r_fT}$. So, the forward prices quoted at a point in time forward price starts at today's stock price and grows at the interest rate as maturity increases.

In this ideal case, holding a forward contract to buy one unit is equivalent to holding one unit of the underlying physical commodity and borrowing the money to pay for it in exchange for repayment of the forward price at the maturity of the forward contract. As the competitive forward price for that maturity changes over time, the holder and issuer experience an economic gain or loss (which may or may not show up in accounting numbers) that generates a cash flow at maturity. Warning: hedge accounting is strange and interesting and almost surely not what you would anticipate.

In-class exercise: forward price of a stock index

A stock index now at 100 is equal to the price of a value-weighted portfolio of stocks that pay no dividends. The one-year interest rate is 10%. The theoretical forward price is 110. Show an arb to exploit the mispricing if the forward price is 105.

	cash now	stock now	cash 1 year out	stock 1 year out
buy/sell forward				
buy/sell stock				
borrow/lend cash				
net				

If there are dividends, would the arb still work?

Forward prices outside the ideal case

The ideal case considered in a previous slide is not an accurate description of the pricing of most forward contracts. If the underlying asset pays dividend or has other "use value," the present value of dividends or use value should be subtracted from today's price before computing the forward price. Conversely, if there are storage costs or insurance costs to be born by the holder of the physical asset, the present value of these costs should be added to the today's price before computing the futures price. For example, if use value comes continuously at a proportional rate u and costs are incurred at a proportional rate c, the forward price should be $F = Se^{(r+c-u)T}$.

Another reason why the forward price might deviate from $F = Se^{r_f T}$ is the effect of harvests or stockouts which imply we cannot short the physical commodity (we cannot borrow units that are needed in production). In agricultural commodities, the potential stockouts occur at predictable times (just before the next harvest), and we can observe downward jumps in the forward price curves at the corresponding maturities. For most commodities, hedging with the wrong maturity of forward contract is risky. The failure of Metalgesselleschaft was due in part to defective oil hedges across maturities.

Problems with forward contracts

- large losses are possible
- you must trust counterparty's credit
- no obvious intermediate exit strategy

Futures contracts

A *futures contract* serves the same basic economic function as a forward contract, but without the drawbacks. Each day a futures contract is "marked to market." If the futures price rises, the holder of a futures contract receives the change in price ("variation"), while if the futures price falls, the holder of a futures contract pays the net change in price. The seller of a futures contract has the opposite treatment, paying the variation when the futures price rises and receiving the variation when the futures price falls.

Marking to market combined with a modest margin account (enough to cover one day's price movement) is a practical resolution of the credit problem. It also makes exit easier, since after marking to market the futures contract is identical to a new futures contract that might be issued the same day.

In interest rates were totally predictable, absence of arbitrage would imply that the futures price is the same as the forward price, and this is usually a good approximation in practice. However, one futures contract represents more action than one forward contract, because the gain or loss due to a price change is realized immediately instead of at the end of the period.

Valuing commodity futures

If the futures price moves as

$$F \quad \begin{array}{l} \nearrow F(1+\delta) \\ \searrow F(1-\delta) \end{array}$$

this means that an investment of 0 paying the change (called variation) is a fairly priced trade:

$$0 \quad \begin{array}{l} \nearrow F\delta \\ \searrow -F\delta \end{array}$$

which is to say that the expected change is zero in the risk-neutral probabilities. For many commodities, this is a good model with delta constant and risk-neutral probabilities of $1/2$ and $1/2$. (We could take the spacing to be unequal to have volatility fall as the price falls, with the necessary adjustment in the probabilities.)

Important point: The expected rate of change of a stock price, under the risk-neutral probabilities, is the riskfree rate r. However, the expected rate of change of a futures price, under the risk-neutral probabilities, is 0. The interest is compensation for using the invested money in the case of the stock. For a futures contract, however, no money has been invested.

Application: synthetic equity and portable alpha

Futures contract are not investments (since no money is paid to enter a futures contract). However, combining stock index futures with investment in bonds is a way of creating a position very similar to holding equities, and is referred to as *synthetic equity*. For example, holding $S\&P$ 500 futures plus a LIBOR investment gives us returns that are very close to the index return but with smaller trading costs than holding the individual stocks in the index.

If we make synthetic equity using an actively managed bond portfolio, this allows us to make use of (intended) superior returns in the bond market but still have an overall equity-like risk profile appropriate to benchmarking to the $S\&P$ 500. This is referred to as "portable alpha:" in the CAPM alpha (or Jensen's alpha) is a measure of superior performance, and we are using futures to move the positive alpha in bonds to create an equity-like portfolio.

Valuing and hedging futures options

Some commodity options are written directly on the underlying and settled by delivery of physicals, and some are written on the underlying and settled in cash at a settlement price. More commonly, the options are written on futures and settled in cash at the futures price. Just like variation margin on futures, the price is a settlement price determined by an exchange employee or a consensus of traders after closing.

Valuing commodity futures options is performed exactly like valuing equity options or interest options, once we have established the risk-neutral probabilities. The simple assumption about moves and risk-neutral probabilities is the one described earlier, although it may be more natural to start with the underlying asset for bond futures options or stock index futures options.

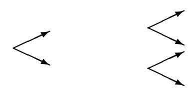
The number of futures to hold is the number needed to give the same difference in value across the two states next period as holding the futures would.

In-class exercise: European call futures option

Consider a two-period binomial model. The short riskless interest rate is fixed at 10% per period. The two-period corn futures price is \$200 today and will go up or down by \$50 each period, with risk-neutral probabilities $1/2$ and $1/2$. What is the price today of a European futures call option with an exercise price of \$200 and maturity two periods from now?

futures

call option

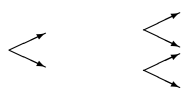


hint: once the tree is drawn, the rest is just like valuing a stock option

In-class exercise: American call futures option

Consider a two-period binomial model. The short riskless interest rate is fixed at 10% per period. The two-period corn futures price is \$200 today and will go up or down by \$50 each period, with risk-neutral probabilities $1/2$ and $1/2$. What is the price today of an American futures call option with an exercise price of \$200 and maturity two periods from now?

futures



call option

puzzle: Why doesn't the usual result about not exercising a call option early apply here?